The Valuation of Deposit Insurance Allowing for the Interest Rate Spread and Early-Bankruptcy Risk

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Abstract

Our study allows for the interest rate spread, defined as the difference between the lending rate and the borrowing rate, and the risk of a bank's early bankruptcy in deriving a closed-form pricing formula for calculating deposit insurance premiums because in practice these two factors significantly influence the expected value of the bank's assets. Also, we use a method that considers the possibility of early bankruptcy to derive new formulas for estimating the necessary parameters. The data from Taiwanese banks are used to illustrate the application of our model. Our empirical results show that the spread of the interest rate is negatively correlated with the premium. This result is consistent with our theoretical inferences. Moreover, premiums associated with a risk of early bankruptcy are always higher than premiums not associated with such risks. The traditional model underprices the premium by 20.81% if it ignores the risk of early bankruptcy. Our results also show that 75% of Taiwanese banks have incentives to risk-shift, and this conclusion applies regardless of whether our pricing model or the traditional pricing model is employed to calculate the deposit insurance premium.

Keywords: Deposit Insurance, Premiums, Interest Rate Spread, Early Bankruptcy, Risk-Shifting

1. Introduction

A well-designed deposit insurance system strengthens the depositor's confidence in financial institutions, and this confidence in turn stabilizes the financial circumstances of a country. Therefore, constructing a good deposit insurance system is crucial for a country's economic development. A fair deposit insurance premium is important for constructing a good deposit insurance system. If the premium is lower than what a bank should pay, the bank is likely to shift its risk to the deposit insurer (Peltzman, 1970; Santomero and Vinso, 1977; Marcus and Shaked, 1984; Ronn and Verma, 1986; Duan, Moreau and Sealey, 1992; Shyu and Tsai, 1999a, b). In contrast, if the calculated premium is too high, the bank may be unwilling to participate in the deposit insurance program at all. These two situations could even result in the collapse of the entire system. Therefore, accurately and fairly valuating deposit insurance premium is important to consider in researching how banks manage risk.

Merton (1977) was the first to derive a formula for pricing risk-based deposit insurance premiums. He argued that this premium is the same as the initial value of a European put option whose strike price and underlying assets are respectively the value of the deposits and the value of the bank's total assets. Numerous researchers have investigated how Merton's valuation framework can be used to more precisely determine deposit insurance premiums (Ronn and Verma, 1986; Allen and Saunders, 1993; Duan and Yu, 1994, Duan and Simonato, 2002; Lee, Lee and Yu, 2005; Chen, Ju, Mazumdar and Verma, 2006; Chuang, Lee, Lin and Yu, 2009). For example, in Merton's model, the value of the bank's assets and the standard deviation of the value of its returns are two important but unobserved parameters. Some papers used the simultaneous equations of the bank's equity and the standard deviation of its equity returns to determine these two parameters (Ronn and Verma,1986; Giammarino, Schwarz and Zechner, 1989). Duan and Yu (1994) used Maximum Likelihood Estimation to estimate them. Recently, some scholars derived a formula for calculating a deposit insurance premium when the stochastic process of the interest rate is included in the model (Duan and Simonato, 2002; Chuang et al, 2009). Pennacchi (1987) made the point that insurance premium depends on whether the deposit insurer is offering a fixed-rate, unlimited-term contract or a variable-rate, limited-term contract. Furthermore, the deposit insurance had considered the default risk of guaranty fund in several studies (Episcopos, 2004).

Regarding the deposit insurance studies, numerous researchers focus the issues on the risk-shifting phenomenon of banks (Santomero and Vinso, 1977; Sharpe, 1978; Marcus and Shaked, 1984; Duan, Moreau and Sealey, 1992; Niinimaki, 2001; Hovakimian, Edward, and Luc, 2003; Gonzalez. 2005; Wagster, 2007; Demirgüç-Kunt, Kane and Laeven, 2008; Gropp, Hakenes and Schnabel, 2011). Most of the researches have used Merton's model to investigate whether a bank shifts its risk. Ronn and Verma (1986) and Giammarino, Schwarz and Zechner (1989) found that bigger banks paid higher premiums than they should have. Sharpe (1978) found that banks that shift their risk to a deposit insurance company do so by using a fixed deposit insurance premium scheme. However, other researchers have found no significant evidence for risk shifting (Santomero and Vinso, 1977; Marcus and Shaked, 1984; and Duan, Moreau and Sealey, 1992).

An important factor on the determination and valuation of deposit insurance premiums is whether the effect of the difference between the lending rate and the borrowing rate (i.e., the interest rate spread) is incorporated in the model. As we know, a commercial bank earns its profits by paying a lower borrowing rate and receiving a higher lending rate. Recently, because of the very competitive financial market, banks have generally either increased the borrowing rate to attract depositors or decreased the lending rate to attract lenders. Both of these actions decrease the interest rate spread. As shown in Figure 1, this spread decreased from the years 2000 to 2010 in the Taiwanese financial market, decreasing the profits of Taiwanese banks. For the commercial banks, bank's profits, and in turn the value of its assets, come from exploiting the interest rate spread. The size of this spread can affect the likelihood of bankruptcy. Merton (1978) who may have been the first to model a “spread” between a bank’s competitive rate of return on assets and deposits. He modeled the spread as a lower-than-competitive interest rate paid to depositors. Pennacchi (1987 and 1987b) allow for a spread in the form of lower-than-competitive deposit interest rates. We also consider the magnitude of the spread when determining the optimal value for a deposit insurance premium in this paper.

In addition, valuation models such as Merton’s are feasible only when a bank goes into bankruptcy at the maturity date of the insurance contract (defined as the auditing date in Merton’s model). Generally, banks are involuntarily closed by the decision of a government regulator or deposit insurer. However, form the financial viewpoint, a bank is bankruptcy if the value of its assets falls below its debt level at any auditing date before the maturity date. We calculate the deposit insurance based on this definition. In such case, using Merton’s model could cause inaccurate calculations of deposit insurance premiums, because it uses only one auditing time to determine the probability of bankruptcy and the expected loss. To accurately determine a fair deposit insurance premium, the model should also consider the risk of early bankruptcy at any of a number of auditing times before the maturity date stipulated in the insurance contract.

When estimating the risk of early bankruptcy, it needs to consider the probability of the early bankruptcy and the recovery given bankruptcy. In traditional research, the first-passage time model has been adopted in many studies aimed at valuating a security when there is an early termination risk (Black and Cox, 1976; Finger, 2002). When using this model to investigate the risk of bankruptcy, the bankruptcy occurs as soon as the bank’s asset is less than the level of its debt (i.e., the first absorption situation). As mentioned by Black and Cox (1976), the first-passage time model cannot be applied directly when the termination payoff is endogenously found to contribute to an optimal-stopping problem. In the papers reporting this use of the first-passage time model, the focus has been exclusively on determining the termination probability (i.e., the probability of the bankruptcy); moreover, the authors usually let this termination payoff (i.e., the recovery given bankruptcy) be uncorrelated with the underlying asset (i.e., bank's asset value). In such cases they must assume that the termination payoff is either an exogenous variable (see Finger, 2002) or an endogenous variable that is uncorrelated with the underlying asset value (see Black and Cox, 1976).

However, when dealing with the risk of bankruptcy in valuating deposit insurance, the factors of probabilities of early bankruptcy and recovery given bankruptcy should be simultaneously determined by the same factor: the bank's asset value. Thus, discussing these two factors separately may result in incorrect pricing of the deposit insurance. We support a new method for accurately pricing a deposit insurance premium, one that takes account of the consideration of a bank's early bankruptcy. The main difference between our model and the first-passage time models is that we do not separate discussion of the probability of early bankruptcy from discussion of a possible recovery given bankruptcy; instead, we let them both be endogenous variables and simultaneously determined by the bank's asset value. In real applications, our specifications yield more accurate pricing than traditional specifications. To the best of our knowledge, ours is the first paper to report derivation of a closed-form deposit insurance pricing formula that let the probability of the early bankruptcy and the recovery given bankruptcy be simultaneously determined by bank's asset value for taking account of the bank's early bankruptcy risk.

Note that our closed-form pricing formula can also be applied to the valuation of an American option because the early exercise probability and the payoff of this option are both determined by the same underlying asset value, as is most likely the case when one uses the specifications of our model. Our formulas give values similar to those reported by Kim (1990) for the valuation of an American option, but ours have the advantage of being easier to understand, thereby making practical application effortless.

In recognition of the fact that both the interest rate spread and the risk of a bank’s early bankruptcy significantly influence the value of the deposit insurance, the author’s goal for this paper is to provide a pricing framework that takes account of these two important factors in accurately valuating deposit insurance premiums and to discuss banks’ risk-shifting behavior.

Moreover, pricing a deposit insurance premium requires estimating the value of the bank’s assets and the standard deviation of the value of its returns. Many researchers have estimated these parameters by equity data (Marcus and Shaked, 1984; Ronn and Verma, 1986). However, if a bank goes bankrupt before the maturity date of the insurance contract, estimates based on formulas such as Ronn and Vermas’ that assume no early bankruptcy may be inaccurate. To more accurately estimate the asset value and the standard deviation of the returns, we need a method that considers the possibility of early bankruptcy. A bank’s stockholders often receive nothing when the bank goes bankrupt. The bank’s equity value, from which the likelihood of early bankruptcy can be inferred, can be determined by employing a down-and-out barrier option with a zero rebate value. We therefore use this option to derive new formulas for estimating the two necessary parameters, the values of which are derived from the first passage time model.

For our empirical analysis, we collected data from 32 banks listed on the Taiwanese stock exchange. After applying the new formulas described above, we calculated a fair, risk-based deposit insurance premium, taking into account the interest rate spread and the risk of early bankruptcy. We discuss below the differences between calculations using our model and the traditional model. The calculation of deposit insurance premium for a bank is used to investigate its risk-shifting incentive. We used the method supported by Duan et al.

(1992) to compare risk-shifting incentives for Taiwanese banks based on the deposit insurance premium calculated by both our pricing model and the traditional pricing model. We also describe another comparison of these results to further elucidate whether the risk-shifting incentive is estimated differently by different models.

The remainder of the paper is organized as follows. Section 2 presents the formula that incorporates the interest rate spread and the early-bankruptcy risk to obtain an approximate price for a deposit insurance premium. Section 3 shows how we utilize the down-and-out barrier option to develop empirical formulas for estimating the value of the bank's assets and the standard deviation of the value of its returns. In this section, we also report the results of our analysis of data from the 32 banks listed on Taiwanese stock exchange, including a comparison between the results calculated by our model and those calculated by the traditional model. Our conclusions are summarized in Section 4.

2. A model for valuating a deposit insurance premium considering the interest rate spread and the likelihood of a bank's early bankruptcy

Let the underlying probability space be labeled (Ω, Λ_F, Q), where Ω is the state space, Λ_F is the filtration generated by the information set $F = (F_t)_{0 \le t \le T}$, and Q is the risk-neutral probability. Let V be the unobserved value of the bank's assets. In Merton (1977), the evolution of V is specified as following a geometric Brownian motion. Given the risk-neutral probability, V can be expressed as,

$$\frac{dV}{V} = (r_f(t) - \delta)dt + \sigma_V dZ_V(t), \qquad (1)$$

where

$r_f(t)$ is the risk-free interest rate;

δ is the dividend yield;

σ_V is the instantaneous standard deviation of bank's value return; and

$Z_V(t)$ is a standard Brownian motion under the risk-neutral measure.

In the above equation, the bank's assets increase by the rate $r_f(t)$ and this value decreases by the dividend yield δ. How much the assets increase depends on what the bank earns from its loans, which, in practice, is determined by the lending rate. Accordingly, Merton's model specifies that the lending rate equal the risk-free interest rate under the risk-neutral measure.

According to Merton (1977), the value of a bank's assets is assumed to grow at the rate $r_f(t)$. Because the debt also grows at this rate, the effect of the interest rate disappears in Merton's formula if it is a constant. However, this assumption is unrealistic. The bank obtains funds from its depositors and then loans these funds to the borrowers. The bank's profits come mainly from managing the deposits and the lending. To earn a profit, the bank's deposit rate must generally be less than its lending rate. The difference between these two rates is defined as the interest rate spread in our model. Because we assume the average cost of the debt paid to the depositors to be the risk-free interest rate, we set the lending rate as $r_f(t)+\kappa(t)$, where $\kappa(t)$ is the interest rate spread under the assumption of risk-neutrality. The value of V can be expressed as:

$$\frac{dV}{V}=(r_f(t)+\kappa(t)-\delta)dt+\sigma_V dZ_V(t)\,. \tag{2}$$

It is worth noting that if we let $\kappa(t)=0$, our model is identical to Merton's pricing formula.

In general, deposit insurance contracts provide depositors with minimal protection against the risk of the bank going bankrupt. We let $B_1(t)$ denote a protected deposit amount at time t, and define the amount of the deposit that is unprotected at time t as $B_2(t)$, which is equivalent to the value of all the debt liabilities other than the insured deposit. Thus, the total value of the deposit at time t can be expressed as $B(t) = B_1(t) + B_2(t)$. The interval $[t, T]$ stands for the period of the deposit insurance contract, with T indicating the maturity date. In Merton's model, the maturity date is specified as an auditing time at which one can judge whether or not the bank is bankrupt. At the maturity date, the total value of the bank's assets is $V(T)$ and the total value of the deposits is $B(T)$.

We choose a money market account to serve as the numeraire asset in our model, expressed as follows:

$$M(t, v) = \exp(\int_t^v r_f(u)du). \tag{3}$$

The process of the money market account is stochastic, because we specify $r_f(t)$ as following a stochastic process. Let the variance of the money market account be denoted as $b^2(t, u)$, its form dependent on the specification of the risk-free interest rate process. For example, if the interest rate is specified as the Vasciek (1977) type, the variance of the money market account can be obtained as the following form (Heath, Jarrow and Morton, 1992):

$$b^2(t, u) = \frac{\sigma_r^2}{a^2}((u-t) - \frac{2}{a}(1 - e^{-a(u-t)}) + \frac{1}{2a}(1 - e^{-2a(u-t)})), \tag{4}$$

where a is the speed of adjustment (a positive constant), and σ_r is the volatility of the risk-free interest rate (a positive constant). Given the specification of the money market account, we have the following equation:

$$B(T) = B(t)e^{\int_t^T r_f(u)du} = B(t)M(t, T). \tag{5}$$

As mentioned by Merton (1977), if $V(T) \geq B(T)$ at the maturity date, the protected amount of the deposit is $B_1(T)$. In contrast, if $V(T) < B(T)$, the bank goes into bankruptcy and the protected amount of the deposit becomes $V(T)\dfrac{B_1(T)}{B(T)}$.[1] The value of the deposit insurance contract at T is then:

$$Max\left[0,\ B_1(T)-V(T)\frac{B_1(T)}{B(T)}\right]. \tag{6}$$

According to Equation (2), the change in the amount of the bank's assets is a random variable that follows a normal distribution. Thus, the deposit insurance premium can be calculated based on the theory of that distribution. In light of the above equation, the contract value can be regarded as a classic European put option with a strike price $B_1(T)$ and an underlying asset value $V(T)\dfrac{B_1(T)}{B(T)}$. The deposit insurance premium, which is denoted as $IP(t,T)$, assuming only one auditing time (at the maturity date) and the given interest rate spread. We can obtain the following result of extending the classical Black-Scholes model (Black and Scholes, 1973):

$$IP(t,T)=B_1(t)N(y_2(t,T))-V(t)e^{(s-\delta)(T-t)}\frac{B_1(t)}{B(t)}N(y_1(t,T)), \tag{7}$$

where

$s=(T-t)^{-1}\int_t^T \kappa(u)du$, defined as the average interest rate spread;

$N(\cdot)$ is the cumulative density of a standard random variable;

$$y_1(t,T)=\frac{\ln(\frac{B(t)}{V(t)})+\eta_1(t,T)}{v(t,T)};$$

$$y_2(t,T)=\frac{\ln(\frac{B(t)}{V(t)})+\eta_2(t,T)}{v(t,T)};$$

[1] For simplified, we assume $B_1(T)$ and $B_2(T)$ have the same seniority.

$$\eta_1(t,T) = -(s-\delta)(T-t) - \frac{1}{2}v^2(t,T);$$

$$\eta_2(t,T) = -(s-\delta)(T-t) + \frac{1}{2}v^2(t,T); \text{ and}$$

$$v^2(t,T) = \int_t^T \left|\sigma_V - b(t,u)\right|^2 du .$$

If we assume a correlation between the return of the value of the bank's assets and a risk-free interest rate as ϑ, we can obtain the following equation (cf. Merton, 1973):

$$v^2(t,T) = \int_t^T (\sigma_V^2 - 2\vartheta\sigma_V(u)b(t,u) + b^2(t,u))du . \tag{8}$$

According to Equation (7), the premium decreases as the interest rate spread increases. We infer from this conclusion that the probability of bankruptcy decreases because the increase in the interest rate spread leads to a rise in the bank's profits. Thus, there is a negative relationship between the interest rate spread and the deposit insurance premium. As mentioned previously, one can easily show that Equation (7) becomes the Merton pricing formula for deposit insurance if $s = 0$ and $r(t)$ is a constant.

The per-monetary-unit deposit insurance premium, $IPP(t,T)$, can be obtained by dividing Equation (7) by $B_1(t)$, as follows:

$$IPP(t,T) = N\left(y_2(t,T)\right) - e^{(s-\delta)(T-t)}\frac{V(t)}{B(t)}N\left(y_1(t,T)\right). \tag{9}$$

In Merton's model, there is only one auditing time for judging whether the bank is bankrupt. However, this specification is not realistic. Because banks are important financial intermediaries, information about whether a bank's assets are less than its debt, which is of concern to many people, must be publicly circulated at a fixed time. As soon as market participants find out a bankrupt signal (i.e., the debt exceeds the assets) for a bank, many people might rush to withdraw their money. In such situation, the bank will go into bankruptcy if there is no help from Government and other financial institutions. Therefore, while the insurance is in force, there are many times at which the bank's financial situation can be checked. These auditing times also offer an opportunity to initiate bankruptcy proceedings if the bank's assets are found to be less than its debt (i.e., $V(t) < B(t)$) at that time. To accurately price deposit insurance, the associated premium, which must reflect the early-bankruptcy risk, should be modeled precisely.

We derive below closed-form formulas for many auditing times under both discrete-time and continuous-time frameworks. We first construct the model under a discrete-time framework. Then, we discuss our continuous-time model, with the time interval approaching zero. We limit the occurrence of a possible bankruptcy to a finite number of auditing times. Let $d = \dfrac{T-t}{\Delta t}$, be the number of auditing time. For example, if $d = 12$, $t = 0$, and $T = 1$ year, the bank is audited at the end of each month. We let t_i represent the date of audit i, we have $t = t_0 < t_1 < \cdots < t_d = T$. $V(t_i)$ and $B(t_i)$, with $i = 0,1,\cdots,d$, denote the total values of the bank's assets and deposits respectively at time t_i. Therefore, at every possible bankruptcy time, there is a corresponding value of the bank's assets, that is, $V(t_0)$, $V(t_1)$, $\cdots, V(t_d)$. Note that the maturity date refers exclusively to the expiration date of the deposit insurance contract. If the contract is perpetual, this date can be specified as an infinite variable.

To take the probability of early bankruptcy into account, calculation of the deposit insurance premium requires the consideration of the expected value of the nonzero random loss (i.e., the payoff shown in Equation (6)). We denote $IP^A(t,T)$ as the deposit insurance premium with the early-bankruptcy risk. At the maturity date, the payoffs of $IP^A(t,T)$ and $IP(t,T)$ are the same; therefore, we have $IP^A(t_{d-1},T) = IP(t_{d-1},T)$. Except for the maturity date, the payoff of the deposit insurance contract at time t_i can be described as follows:

The payoff of the deposit insurance contract at time t_i

$$=\begin{cases}\text{The live value of the deposit insurance contract,} & \text{if } V(t_i)\geq B(t_i)\\ \text{The termination value of deposit insurance contract,} & \text{if } V(t_i)<B(t_i)\end{cases}. \tag{10}$$

Therefore, at time t_{d-2} the amount of time until maturity is $2\Delta t$. Thus, $IP^A(t_{d-2},T)$ can be expresses as follows:

$$IP^A(t_{d-2},T)=E[\frac{IP^A(t_{d-1},T)}{M(t_{d-2},t_{d-1})}(1-I_{D(t_{d-1})})\mid F_{t_{d-2}}]$$
$$+E[\frac{B_1(t_{d-1})(1-\frac{V(t_{d-1})}{B(t_{d-1})})}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\mid F_{t_{d-2}}], \tag{11}$$

where $E[\cdot\mid F_t]$ represents an expected operator conditional on information set F_t with a risk-neutral probability, and $I_{D(t_{d-1})}$ is an indicator function. This gives us $I_{D(t_{d-1})}=1$ if $D(t_{d-1})=\{V(t_{d-1})<B(t_{d-1})\}$; otherwise, and $I_{D(t_{d-1})}=0$. The first and second terms of the right hand side of Equation (11) are the expected survival and termination values respectively of the deposit insurance contract at time t_{d-2}. Based on the previous specifications, $IP^A(t_{d-2},T)$ can be described as follows (see Appendix A):

$$IP^A(t_{d-2},T)=IP(t_{d-2},T)+E[\frac{(s-\delta)\Delta t\,V(t_{d-1})\frac{B_1(t_{d-1})}{B(t_{d-1})})}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\mid F_{t_{d-2}}]. \tag{12}$$

We define the second term of the right side of Equation (12) as the premium associated with an early bankruptcy risk (i.e., the early-bankruptcy premium). At each time point, $IP^A(\cdot,T)$ is the sum of the corresponding $IP(\cdot,T)$ plus the early-bankruptcy premium. As shown by Kim (1990), if we work backwards recursively, the value of the deposit insurance contract with the early-bankruptcy risk at time t can be described as follows (see also Appendix A):

$$IP^A(t,T) = IP(t,T) + \sum_{i=0}^{d} (s-\delta)\Delta t\, e^{(s-\delta)(t_i-t)} V(t) \frac{B_1(t)}{B(t)} N(y_1(t,t_i))) . \tag{13}$$

The per-monetary-unit deposit insurance premium including the early bankruptcy risk at time t, or $IPP^A(t,T)$, can be represented as follows:

$$IPP^A(t,T) = IPP(t,T) + \sum_{i=0}^{d} (s-\delta)\Delta t\, e^{(s-\delta)(t_i-t)} \frac{V(t)}{B(t)} N(y_1(t,t_i))) . \tag{14}$$

We now move from the discrete-time framework to the continuous-time framework, where we let Δt tend toward zero (i.e., infinite auditing times). Equations (13) and (14) can then be rewritten as follows:

$$IP^A(t,T) = IP(t,T) + \int_t^T (s-\delta)\, e^{(s-\delta)(u-t)} V(t) \frac{B_1(t)}{B(t)} N(y_1(t,u)))du ; \tag{15}$$

and

$$IPP^A(t,T) = IPP(t,T) + \int_t^T (s-\delta)\, e^{(s-\delta)(u-t)} \frac{V(t)}{B(t)} N(y_1(t,u)))du . \tag{16}$$

To be mentioned, although we model the risk to the bank's assets to include only one Brownian motion as shown in Equation (2), the loss (i.e., $V(t) - B(t)$) incurred from going into bankruptcy is not always zero even under the continuous-time framework. Given the properties of Brownian motion, a large increase or a large decrease in a bank's assets may occur at next time point in theory. Therefore, a sudden and large decrease in a bank's assets may cause the value of these assets to drop below the value of the bank's debt (i.e., $V(t) < B(t)$) even on the continuous-time framework.

Equation (16) shows that if a bank's earnings from the interest rate spread is larger than the payout rate (i.e., the dividend rate) and the possibility of early bankruptcy is taken into account, the bank should pay a larger deposit insurance premium using our model than using the traditional model. On the other hands, the deposit insurance premium is smaller using our model than using the traditional model if the interest rate spread is smaller than the dividend rate. These results are reasonable because the deposit insurance premium is the insurer's expected loss. If the drift term of the asset return is positive (e.g., the earnings are greater than the payouts), the expected value of the bank's assets increases during the contract period. In this situation, the likelihood of bankruptcy is determined only by the volatility of the assets. The expected loss calculated from our formula is larger than that calculated from the traditional formula, because our formula takes account of many possible losses for the insurer before the maturity date of the contract. By contrast, if the bank's earnings are less than its payouts, the likelihood of bankruptcy increases not only because of the volatility the bank's assets, but also because of the decrease in their value. Because the drift term of the asset return is a decreasing function of time, the expected value of the assets eventually falls below the value of the debt (i.e., bankruptcy) if there is a long time before the contract reaches maturity. The insurer's loss calculated from a model that includes many auditing times (i.e., our model) is smaller than that calculated by a model that includes only one auditing time (i.e., the traditional model). This is because our model identifies the date of the bankruptcy as earlier than the traditional model can, and it therefore can forestall an enlargement of the loss. Thus, $IPP^{A}(t,T) \leq IPP(t,T)$ when $s \leq \delta$.

When valuating a deposit insurance premium, the bank's total assets and the standard deviation of the value of its returns must be estimated. Some authors used equity data to estimate these two parameters (see, Marcus and Shaked,1984; and Ronn and Verma, 1986). For simplicity, we let the interest rate be a constant or a deterministic variable. The following simultaneous equations reflect the method introduced by Ronn and Verma (1986):

$$E(t) = V(t)e^{(s-\delta)(T-t)}N(x_1) - \rho B(t)N(x_2) \text{, and} \tag{17}$$

$$\sigma_E = \frac{V(t)}{E(t)}\frac{\partial E(t)}{\partial V(t)}\sigma_V = \frac{V(t)}{E(t)}e^{(s-\delta)(T-t)}N(x_1)\sigma_V , \tag{18}$$

where

$E(t)$ is the value of a bank's equity at time t;

σ_E is the standard deviation of a bank's equity return;

ρ is the level of declaring bankruptcy;

$$x_1 = \frac{\ln(\frac{V(t)}{\rho B(t)}) - \eta_1(t,T)}{\sigma_V^2(T-t)}; \text{ and}$$

$$x_2 = \frac{\ln(\frac{V(t)}{\rho B(t)}) - \eta_2(t,T)}{\sigma_V^2(T-t)}.$$

Equation (17) shows that the equity value equals the present value of the surplus assets after the debt is paid off at maturity. The payoff of the equity is $(V(T) - \rho B(T))^+$ at the maturity date. Equation (18) gives the standard deviation of the value of the bank's returns, derived using Ito's Lemma.

However, the estimates of the bank's assets and the standard deviation of its returns calculated from the above equations may be inaccurate, because they do not take into account the probability of a bankruptcy prior to maturity. We therefore revise the formulas to address this problem. As is well known, a bank's stockholders often receive nothing when the bank goes bankrupt. Therefore, the bank's equity value can be regarded as the value of a down-and-out barrier option with zero rebate value. Thus, we adopt the first passage time model to derive the revised formulas. Based on this model, the closed-form formula for the equity can be expressed as follows (see Musiela and Rutkowski, 2002):

$$E(t) = V(t)e^{(s-\delta)(T-t)}[N(x_1) - (\frac{\rho B(t)}{V(t)})^{2(s-\delta)v^{-2}(t,T)+1} N(h_1)]$$

$$- \rho B(t)[N(x_2) - (\frac{\rho B(t)}{V(t)})^{2(s-\delta)v^{-2}(t,T)-1} N(h_2)], \qquad (19)$$

where $h_1 = \dfrac{\ln \dfrac{\rho B(t)}{V(t)} - \eta_1(t,T)}{\sigma_V^2 (T-t)}$, and $h_2 = h_1 - \sigma_V^2 (T-t)$.

Moreover, using Ito's Lemma, the standard deviation of the equity value is obtained as follows:

$$\sigma_E = \frac{V(t)}{E(t)} \Phi(t) \sigma_V , \tag{20}$$

where

$$\Phi(t) = \frac{\partial E(t)}{\partial V(t)}$$

$$= e^{(s-\delta)(T-t)} [N(x_1) + 2(s-\delta)(\sigma_V^2 (T-t))^{-1} (\frac{\rho B(t)}{V(t)})^{2(s-\delta)(\sigma_V^2 (T-t))^{-1}+1} N(h_1)]$$

$$-(2(s-\delta)(\sigma_V^2 (T-t))^{-1} - 1)(\frac{\rho B(t)}{V(t)})^{2(s-\delta)(\sigma_V^2 (T-t))^{-1}} N(h_2) .$$

We then use these formulas to estimate the value of the bank's assets and the standard deviation of its returns. One can obtain more reasonable deposit insurance premiums by adopting the parameters estimated from Equations (19) and (20) to our pricing formula.

As noted in their publications, scholars who follow traditional methods are interested in determining the likelihood that banks will transfer their risks to deposit insurance companies if they think the premiums are too low (Santomero and Vinso, 1977; Sharpe, 1978; Marcus and Shaked, 1984; Ronn and Verma, 1986; Giammarino, Schwarz and Zechner, 1989; Duan, Moreau and Sealey, 1992; and Shyu and Tsai, 1999a, b). One may also be interested to learn whether the findings regarding a bank's risk-shifting incentive are the same when deposit insurance premiums are determined by our pricing model and by the traditional pricing model.

Duan et al. (1992) published the following formula for discussing a bank's risk-shifting incentive:

$$\Delta\frac{B(t)}{V(t)} = a_1 + b_1\Delta\sigma_V^2(t) + \varepsilon_1 \text{, and} \tag{21}$$

$$\Delta IPP(t) = a_2 + b_2\Delta\sigma_V^2(t) + \varepsilon_2 \,, \tag{22}$$

where a and b are estimated parameters and ε is the residual. The null hypotheses are:

$H_0 : b_1 \le 0$ and $H_0 : b_2 \le 0$.

According to Duan et al. (1992), the rejection of $b_1 \le 0$ is a necessary but not sufficient condition for the absence of a risk-shifting incentive in the banking system. Thus, if b_1 is strongly negative, b_2 should also take a negative value. If $b_1 \le 0$ is rejected, an increase in the assets risk does not prohibit an increase in leveraging. If $b_2 \le 0$ is rejected, it is concluded that the bank has a risk-shifting incentive. We also use these concepts to exam the bank's risk-shifting incentive in this paper.

3. Empirical methods and results

We use data from banks listed on the Taiwanese stock exchange to illustrate the application of our model. At the end of 2010, 32 banks were publicly offered in Taiwan. Annual figures, including the banks' equities, debts, cash dividends, and stock prices, were taken from the TEJ Databank.[2] The sample period is from 1980 to 2010. The standard deviation of the annual equity return for each bank is calculated from its daily stock prices during the corresponding year. Some banks were integrated during the sample period, and their data are missing from the TEJ Databank. For estimating the insurance premiums of these integrated banks, we use only data that can be observed before the integrated date. During the sample period, the Taiwanese government encouraged financial institutions to merge, creating a single financial holding company. For these banks, we use the holding company's stock price as the bank's stock price after the date of the merger. The main reason for doing so is that only the holding company's stock price was available after that time. If the stock prices of both the bank and the holding company are simultaneously shown for a year during the year of the merger, we use the average of the two stock prices for that year. Table 1 presents descriptive statistics for the sample, specifically, the mean and standard deviation of the equity, mean debt, mean cash dividends, and the sample number for each bank.

<Insert Table 1 Here>

Next, we compare results from the different models. The values of the assets and the standard deviations of the returns are estimated from Equations (19) and (20), hereafter referred to as the FPT formulas, and from Equations (17) and (18) of Ronn and Verma (1986), hereafter referred to as the RV formulas. We use the grid search method to estimate the values of the assets and the standard deviations of the returns. In this method, we generate 1000 possible asset values within the range $[A_0(t), A_0(t)+2B(t)]$, where $A_0(t)=E(t)+B(t)$, and 200 corresponding standard deviations within the range $[0.01\times\sigma_E, 2\times\sigma_E]$.[3] The optimal estimated values of $V(t)$ and σ_V are based on the minimum values of the sum of square error, which is defined as follows:

$$\text{sum of squre error} = \left(\frac{\hat{E}(t)-E(t)}{E(t)}\right)^2 + \left(\frac{\hat{\sigma}_E^2(t)-\sigma_E^2(t)}{\sigma_E^2(t)}\right)^2, \tag{23}$$

where $\hat{E}(t)$ and $\hat{\sigma}_E^2(t)$ are the respective mean and standard deviation of the equity, calculated from the FPT and RV formulas, given different values of $V(t)$ and σ_V.

We let $s=0.02$, $t=0$, $T=1$ year, and $\rho=0.97$, the latter based on Ronn and Verma

[2] TEJ is a well-known databank containing financial information on Taiwan.

[3] In general, the volatility of asset is smaller than the volatility of equity in empirical results. This range is used for saving the estimation time. One can also use a larger range for the estimation.

(1986). Table 2 shows the estimated values of the bank's assets and the standard deviations of its returns, based on the FPT and RV formulas.

<Insert Table 2 Here>

Table 2 shows that most of the estimates from the RV formulas are equal to or less than those from the FPT formulas. For example, the assets of Bank 1 are estimated as 891.250 billion from the FPT formulas and 888.840 billion from the RV formulas. Likewise, the estimated standard deviation of the returns is 4.418% from the FPT formulas and 4.330% from the RV formulas. However, in some cases (e.g., Banks 12, 13, 21, 22, 31, and 32) the two formulas give the same estimates. Therefore, we conclude that the model that takes account of a possible early bankruptcy gives equal or lower estimates for the assets and returns than the model that does not take this into account. In theory, we can infer that the lower the value of the assets, the higher the deposit insurance premium; but the lower the standard deviation of the returns, the lower the deposit insurance premium. Therefore, we cannot determine from the results based on the FPT formulas whether deposit insurance premiums increase or decrease under the tested circumstances.

Table 3 shows these estimated deposit insurance premiums, based on the above parameters as estimated by the various formulas. The results include premium estimates incorporating the interest rate spread alone (IPP from Equation (9)) and estimates incorporating both the interest rate spread and the early-bankruptcy risk (IPP^A from Equation (16)). For example, the IPP values for Bank 1 (Chang Hwa Bank), calculated in accordance with the parameters estimated by the FPT and RV formulas, are 3.0400×10^{-3} and 3.6301×10^{-3} respectively; the corresponding IPP^A values are 3.9929×10^{-3} and 5.0657×10^{-3}. Table 3 also shows that IPP^A is always higher than the corresponding IPP, regardless of the parameters estimated by the FPT and RV formulas. The means of the IPP/IPP^A ratios (IPP divided by IPP^A) are nearly 74% for the FPT formulas and 71% for the RV formulas. The values of deposit insurance premium in the second and fifth columns are calculated using both our model (i.e., the FPT formulas and our pricing formula) and the traditional model (i.e., the RV formulas and the European pricing formula). Using the values in these two columns, we find that the mean IPP/IPP^{A} ratio is 79.19%. In other words, on average, deposit insurance premiums for Taiwanese banks are underestimated by 20.81% (i.e., 1-79.19%) using the traditional pricing formula. This result suggests that insurers who use the traditional model are likely to conclude that they have insufficient funds to cover the bank's default risk, because they ignore the possibility of early bankruptcy in calculating the deposit insurance premium.

<Insert Table 3 Here>

As mentioned in a previous section, our model indicates that if a bank's interest rate spread is smaller than its dividend yield, the deposit insurance premium using our model is larger than using the traditional model. Figure 2 shows the empirical evidence for the sensitivity of deposit insurance premiums to changes in the interest rate spread. We take Bank 1 as our example. According to Table 2, its average estimated dividend yield, computed by the FPT formulas, is 0.16%.[4] Therefore, we let the interest rate spread range from 0.1% to 1%. As shown in Figure 2, both IPP and IPP^{A} are negatively correlated with the interest rate spread. In other words, if a bank's earnings increase because of the interest rate spread, the deposit insurance premium decreases. We have also proven that if the interest rate spread is smaller (larger) than the dividend yield, IPP^{A} is smaller (larger) than IPP. Figure 1 also shows that the IPP^{A} curve is flatter than the IPP curve. In other words, compared with a model that takes into account only the likelihood of bankruptcy at the maturity date, the sensitivity of the deposit insurance premium to the change in the interest rate spread is less under the model that takes account of a possible early bankruptcy.

[4] The estimated dividend yield equals the cash dividend divided by the estimated value of the assets.

<Insert Figure 2 Here>

We examine asset values and their standard deviations, both estimated by the FPT formulas, to test the risk-shifting incentive of Taiwanese banks using the Duan et al. (1992) method. As shown in Table 4, there is no bank for which the null hypothesis ($b_1 \le 0$) is rejected. Thus, we may conclude that for all Taiwanese banks an increase in leveraging is prohibited by an increase in the assets risk. We infer from this that bank regulators have effectively restrained banks' leverage activities when the bank's assets are at high risk. On the other hand, the other null hypothesis ($b_2 \le 0$) is rejected for 23 of the 32 banks, regardless of whether IPP or IPP^A is used; by this criterion, 75% of Taiwanese banks have a risk-shifting incentive. The explanation for these results may be that the deposit insurance premium is fixed for Taiwanese banks. In addition, the inferences about risk-shifting incentives for banks are the same regardless of which pricing model is used to determine the deposit insurance premium.

<Insert Table 4 Here>

4. Conclusion

The deposit insurance system plays an important role in a country's financial market. A well-designed system can facilitate the country's economic development because it can prevent an economic crisis from spreading if the banks go bankrupt. The maintenance of a good deposit insurance system depends strongly on the determination of a fair deposit insurance premium. Therefore, how best to evaluate a deposit insurance premium receives a lot of attention, not only from deposit insurance companies but also from academic researchers. The main objective of the present study was to provide the accurate closed-form pricing formula and the optimal empirical methods for determining fair deposit insurance premiums.

In practice, the interest rate spread affects a bank's earnings, and this will in turn further influence the value of the bank's assets and the likelihood of it going bankrupt. In addition, it must also be kept in mind that a bank can go bankrupt prior to the maturity date of the deposit insurance contract. These two factors should influence whether a deposit insurance premium is accurately valuated. Therefore, in this study we incorporated these two factors in our model for deriving an approximate pricing formula for the deposit insurance premium. Our consideration of the early-bankruptcy risk also led us to introduce new empirical formulas derived from the first passage time model to estimate the necessary parameters, namely, the value of the bank's assets and the standard deviation of its returns. Therefore, our model should help deposit insurance institutions more precisely determine fair premiums.

We collected data from the 32 banks listed on the Taiwan stock exchange to illustrate the operation of our model. We have shown that, regardless of the theoretical inferences or the empirical evidence, deposit insurance premiums are negatively correlated with the interest rate spread. We also used numerical results to prove that the sensitivity of deposit insurance premiums to changes in the interest rate spread is lessened if the premium is priced by a model that allows for the possibility of early bankruptcy.

Our study also compared results that were calculated based on the parameters estimated by our formulas (FPT formulas) and by Ronn and Verma's (1986) formulas (RV formulas). Our results indicate that most of the values estimated by the RV formulas are lower than those estimated by the FPT formulas. In other words, the RV formulas can underestimate the value of a bank's assets as well as its risk. In addition, deposit insurance premiums based on the possibility of early bankruptcy (IPP^A) are always higher than premiums not based on a consideration of this risk (IPP), regardless of whether the FPT or RV formulas are used to estimate the parameters. Our results indicate that deposit insurance premiums calculated by the model without taking account of the early bankruptcy risk are underpriced by 20.81%. If a deposit insurance company prices its premiums using the traditional model and ignores the risk of early bankruptcy, the premiums will be too low.

We also were concerned with the question of how likely it is that banks will shift their risks to deposit insurance companies. We conclude from our analyses that an increase in asset risk limits the increase in capital leveraging for all Taiwanese banks. Thus, bank regulators must restrain this leveraging when asset risks increase. Nonetheless, we found that 75% of Taiwanese banks engage in risk-shifting behavior, perhaps because the Taiwanese government uses a fixed deposit insurance system. These conclusions about risk-shifting behavior apply regardless of whether the deposit insurance premiums are calculated from the traditional pricing model or our pricing model. Currently, there is only the value of deposit insurance for each bank throughout our model and the sample. For future research, it may be interesting to see how they vary over the time and whether their dynamics match well with important news events.

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Appendix A

This appendix gives the derivation of Equation (12). According to Equation (11), we have the following:

$$IP^{A}(t_{d-2},T)=E[\frac{IP^{A}(t_{d-1},T)}{M(t_{d-2},t_{d-1})}(1-I_{D(t_{d-1})})\,|\,F_{t_{d-2}}]$$

$$+E[\frac{B_1(t_{d-1})(1-\frac{V(t_{d-1})}{B(t_{d-1})})}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\,|\,F_{t_{d-2}}]$$

$$=E[\frac{IP(t_{d-1},T)}{M(t_{d-2},t_{d-1})}(1-I_{D(t_{d-1})})\,|\,F_{t_{d-2}}]$$

$$+E[\frac{B_1(t_{d-1})(1-\frac{V(t_{d-1})}{B(t_{d-1})})}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\,|\,F_{t_{d-2}}]$$

$$=IP(t_{d-2},T)+E[\frac{B_1(t_{d-1})(1-\frac{V(t_{d-1})}{B(t_{d-1})})}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\,|\,F_{t_{d-2}}]$$

$$-E[\frac{IP(t_{d-1},T)}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})})\,|\,F_{t_{d-2}}].$$

By substituting $IP(t_{d-1},T)$ from Equation (7) into the above equation, we get the following:

$$IP^{A}(t_{d-2},T)=IP(t_{d-2},T)+(E[\frac{B_1(t_{d-1})(1-E[I_{D(t_d)}\,|\,F_{t_{d-1}}])}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\,|\,F_{t_{d-2}}] \tag{A1}$$

$$-E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}(1-e^{-(\delta-s)\Delta t}E[I_{D(t_d)}\,|\,F_{t_{d-1}}])}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\,|\,F_{t_{d-2}}]).$$

If we let $e^{-(\delta-s)\Delta t}=1-(\delta-s)\Delta t+((\delta-s)\Delta t)^2+\cdots$, we have

$$1-e^{-(\delta-s)\Delta t}E[I_{D(t_d)}\,|\,F_{t_{d-1}}]=1-(1-(\delta-s)\Delta t+\cdots)E[I_{D(t_d)}\,|\,F_{t_{d-1}}]$$

$$=1-E[I_{D(t_d)}\,|\,F_{t_{d-1}}]+((\delta-s)\Delta t)E[I_{D(t_d)}\,|\,F_{t_{d-1}}]+O(\Delta t)$$

where $O(\Delta t)$ is a function of the higher order of Δt. If we let Δt tend toward zero, we have $O(\Delta t)\cong 0$.

Equation (A1) can be rewritten as follows:

$$IP^{A}(t_{d-2},T)=IP(t_{d-2},T)+(E[\frac{B_1(t_{d-1})(E[1-I_{D(t_d)}\,|\,F_{t_{d-1}}])}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\,|\,F_{t_{d-2}}]$$

$$-E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}((E[1-I_{D(t_d)}\mid F_{t_{d-1}}])+(\delta-s)\Delta t)E[I_{D(t_d)}\mid F_{t_{d-1}}]))}{M(t_{d-2},t_{d-1})}I_{D(t_{d-1})}\mid F_{t_{d-2}}]),$$

Because $t_{d-2}<t_{d-1}<t_d$, using expectation theory, we have:

$$E[\frac{B_1(t_{d-1})}{M(t_{d-2},t_{d-1})}E[1-I_{D(t_d)}\mid F_{t_{d-1}}]I_{D(t_{d-1})}\mid F_{t_{d-2}}]=E[\frac{B_1(t_{d-1})}{M(t_{d-2},t_{d-1})}(1-I_{D(t_d)})I_{D(t_{d-1})}\mid F_{t_{d-2}}];$$

$$E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}}{M(t_{d-2},t_{d-1})}E[1-I_{D(t_d)}\mid F_{t_{d-1}}]I_{D(t_{d-1})}\mid F_{t_{d-2}}])$$

$$=E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}}{M(t_{d-2},t_{d-1})}(1-I_{D(t_d)})I_{D(t_{d-1})}\mid F_{t_{d-2}}]),\text{ and}$$

$$E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}(\delta-s)\Delta t}{M(t_{d-2},t_{d-1})}E[I_{D(t_d)}\mid F_{t_{d-1}}]I_{D(t_{d-1})}\mid F_{t_{d-2}}])$$

$$=E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}(\delta-s)\Delta t}{M(t_{d-2},t_{d-1})}I_{D(t_d)}I_{D(t_{d-1})}\mid F_{t_{d-2}}]).$$

According to our specifications, $I_{D(t_{d-1})}$ means that the bank defaulted at time t_{d-1}. If the bank has defaulted at time t_{d-1}, it will still default at time t_d. That is $I_{D(t_d)}=1$ when $I_{D(t_{d-1})}=1$. Therefore, we have $(1-I_{D(t_d)})I_{D(t_{d-1})}=0$ no matter for $I_{D(t_{d-1})}=1$ or 0. This inference is reasonable because $(1-I_{D(t_d)})I_{D(t_{d-1})}$ means that the bank was in default at time t_{d-1} but not at time t_d, a logical impossibility. We therefore have:

$$E[\frac{B_1(t_{d-1})}{M(t_{d-2},t_{d-1})}(1-I_{D(t_d)})I_{D(t_{d-1})}\mid F_{t_{d-2}}]=0;\text{ and}$$

$$E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}}{M(t_{d-2},t_{d-1})}(1-I_{D(t_d)})I_{D(t_{d-1})}\mid F_{t_{d-2}}])=0$$

In addition, we have $I_{D(t_{d-1})}I_{D(t_d)}$ equals to 1 and 0 when $I_{D(t_{d-1})}$ equals to 1 and 0, respectively. We conclude that $I_{D(t_{d-1})}I_{D(t_d)}=I_{D(t_{d-1})}$. We therefore have:

$$E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}(\delta - s)\Delta t}{M(t_{d-2},t_{d-1})} I_{D(t_d)} I_{D(t_{d-1})} \mid F_{t_{d-2}}])$$

$$= E[\frac{B_1(t_{d-1})\frac{V(t_{d-1})}{B(t_{d-1})}(\delta - s)\Delta t}{M(t_{d-2},t_{d-1})} I_{D(t_{d-1})} \mid F_{t_{d-2}}])$$

In the light of the previous results, Equation (A1) can be rewritten as follows:

$$IP^A(t_{d-2},T) = IP(t_{d-2},T) + E[\frac{(s-\delta)\Delta t V(t_{d-1})\frac{B_1(t_{d-1})}{B(t_{d-1})})}{M(t_{d-2},t_{d-1})} I_{D(t_{d-1})} \mid F_{t_{d-2}}].$$

This is Equation (12).

To obtain Equation (13), by working backwards recursively from Equation (A1) and again defining Δt tending toward zero, we have

$$IP^A(t,T) = IP(t,T) + \sum_{i=0}^{d} E[\frac{(s-\delta)V(t_i)\frac{B_1(t_i)}{B(t_i)}}{M(t,t_i)} I_{D(t_i)} \mid F_t]. \tag{A2}$$

We then have

$$E[\frac{(s-\delta)V(u)\frac{B_1(u)}{B(u)}}{M(t,u)} I_{D(u)} \mid F_t] = (s-\delta)\frac{B_1(t)}{B(t)} E[\frac{V(u)}{M(t,u)} I_{D(u)} \mid F_t]$$

$$= (s-\delta)\, e^{(s-\delta)(u-t)} V(t)\frac{B_1(t)}{B(t)} N(y_1(t,u)).$$

By substituting this result into Equation (A2), we get Equation (13).

Table 1: Basic information about the bank sample

Symbol	Description	Equity (billion)	Equity Std (%)	Debt (billion)	Dividend (billion)	Sample Number
Bank 1	Chang Hwa Bank	48.574	43.299	839.430	1.448	27
Bank 2**	First Bank	64.967	41.992	968.030	2.098	27
Bank 3**	Hua Nan Bank	55.745	44.227	905.450	1.861	29
Bank 4**	China Development Industrial Bank	48.398	44.696	43.847	1.378	30
Bank 5**	Mega International Commercial Bank	40.818	40.217	362.500	1.128	22
Bank 6	Standard Chartered Bank (Taiwan)	10.709	42.127	179.260	0.101	24
Bank 7	King's Town Bank	7.544	42.569	93.114	0.011	28
Bank 8*	Kaohsiung Business Bank	108.530	52.307	55.669	0.009	18
Bank 9*	Taitung Business Bank	7.733	54.287	25.740	0.001	23
Bank 10	Taichung Commercial Bank	16.155	43.592	167.400	0.051	27
Bank 11**	Chinatrust bank	69.951	39.823	825.480	2.510	20
Bank 12**	Cathay United Bank	165.120	37.726	908.730	3.287	13
Bank 13**	Taipei Fubon Bank	77.592	34.611	860.080	2.292	12
Bank 14*	The Chinese Bank	12.378	29.813	170.020	0.026	12
Bank 15	Taiwan Business Bank	38.178	40.665	968.600	0.214	13
Bank 16	Bank of Kaohsiung	5.522	37.561	161.610	0.216	13
Bank 17	Cosmos Bank	131.160	42.851	169.460	0.020	16
Bank 18	Union Bank of Taiwan	18.130	33.742	224.460	0.045	16
Bank 19**	Bank of SinoPac	34.120	36.040	496.500	0.800	15
Bank 20**	E.SUN Commercial Bank	28.118	31.544	419.280	0.540	16

Note: This table gives data from the 32 banks listed on the Taiwanese stock exchange. The sample period was from 1980 to 2010. The first column displays the bank symbols and the second column shows their names. The third and fifth columns give the means of the banks' equity and debt values during the sample period. The fourth column gives the standard deviations of the banks' equity returns during the sample period, calculated from the daily equity values from 1980 to 2010. The sixth column gives the means for the banks' cash dividends. The final column gives the sample number of each bank. "*" denotes that the banks have been integrated during sample period. "**" denotes that the banks became financial holding companies.

Table 1: Basic information about the bank sample (continuously)

Symbol	Description	Equity (billion)	Equity Std (%)	Debt (billion)	Dividend (billion)	Sample Number
Bank 21**	Yuanta Bank	20.609	37.587	224.130	0.063	15
Bank 22**	TaiShin Bank	18.507	40.791	190.560	0.111	7
Bank 23	Far Eastern International Bank	17.819	39.171	223.580	0.185	16
Bank 24*	Chung Shing Bank	193.420	80.856	148.130	0.058	7
Bank 25	Public Bank	17.730	39.133	240.090	0.037	15
Bank 26	Entie Commercial Bank	30.034	39.338	205.790	0.094	16
Bank 24	Industrial bank of Taiwan	15.009	50.428	85.775	0.693	7
Bank 28*	Bowa Bank	18.504	44.090	147.620	0.016	12
Bank 29**	JihSun Bank	20.973	40.969	189.340	0.000	15
Bank 30*	Bank of Overseas Chinese citi bank	18.488	49.510	258.110	0.000	9
Bank 31**	Taiwan Cooperative Bank	72.674	34.270	2263.000	2.164	7
Bank 32	Bank of Taipei	1.834	64.243	38.254	0.074	4

Note: This table gives data from the 32 banks listed on the Taiwanese stock exchange. The sample period was from 1980 to 2010. The first column displays the bank symbols and the second column shows their names. The third and fifth columns give the means of the banks' equity and debt values during the sample period. The fourth column gives the standard deviations of the banks' equity returns during the sample period, calculated from the daily equity values from 1980 to 2010. The sixth column gives the means for the banks' cash dividends. The final column gives the sample number of each bank. * denotes that the banks have been integrated during sample period. ** denotes that the banks became financial holding companies.

Table 2: Estimates of each bank's assets and the standard deviations of its returns (Value Std)

Symbol	FPT formulas		RV formulas	
	Bank value (billion)	Value Std (%)	Bank value (billion)	Value Std (%)
Bank 1	891.250	4.418	888.840	4.330
Bank 2	1035.400	4.415	1034.000	4.199
Bank 3	964.940	4.768	962.100	4.423
Bank 4	98.869	15.232	96.492	14.237
Bank 5	404.070	4.661	403.680	4.315
Bank 6	190.990	4.213	190.150	4.213
Bank 7	101.000	4.630	100.750	4.257
Bank 8	164.250	18.468	164.250	18.538
Bank 9	33.534	11.812	33.498	10.050
Bank 10	184.030	4.880	183.720	4.419
Bank 11	896.250	4.238	896.250	3.982
Bank 12	1074.800	4.447	1074.800	4.447
Bank 13	938.530	3.461	938.530	3.461
Bank 14	182.810	3.104	182.570	3.104
Bank 15	1011.300	4.067	1007.700	4.067
Bank 16	168.380	3.756	167.300	3.756
Bank 17	300.790	10.555	300.790	10.772
Bank 18	242.880	3.594	242.820	3.594
Bank 19	531.120	3.704	531.120	3.604
Bank 20	448.400	3.154	447.820	3.154

Note: The first column displays the bank symbols. The second and third columns give the estimated values of the banks' assets and the standard deviations of their returns (denoted as Value Std), calculated using the FPT formulas (expressed in Equations (19) and (20)). The fourth and fifth columns give the corresponding estimates using the RV formulas (expressed in Equations (17) and (18)). The interest rate spread is set as 0.02.

Table 2: Estimates of each bank's assets and the standard deviations of its returns (continuously)

Symbol	FPT formulas		RV formulas	
	Bank value (billion)	Value Std (%)	Bank value (billion)	Value Std (%)
Bank 21	244.960	3.862	244.960	3.862
Bank 22	209.260	4.317	209.260	4.317
Bank 23	241.620	4.020	241.620	3.917
Bank 24	341.700	19.077	341.700	19.322
Bank 25	258.090	4.017	258.060	4.017
Bank 26	236.030	4.778	236.030	4.824
Bank 24	100.870	7.129	100.870	7.356
Bank 28	166.280	5.635	166.280	5.697
Bank 29	210.760	4.350	210.500	4.237
Bank 30	276.850	5.149	276.850	5.149
Bank 31	2339.300	3.427	2337.900	3.427
Bank 32	40.127	6.424	40.127	6.424

Note: The first column displays the bank symbols. The second and third columns give the estimated values of the banks' assets and the standard deviations of their returns (denoted as Value Std), calculated using the FPT formulas (expressed in Equations (19) and (20)). The fourth and fifth columns give the corresponding estimates using the RV formulas (expressed in Equations (17) and (18)). The interest rate spread is set as 0.02.

Table 3: Estimates of the deposit insurance premiums of per deposit amount

Symbol	FPT formulas			RV formulas		
	IPP **($)**	IPP^A **($)**	Ratios (%)	IPP **($)**	IPP^A **($)**	Ratios (%)
Bank 1	3.0400	3.9929	0.7614	3.6301	5.0657	0.7166
Bank 2	2.9405	3.7127	0.7920	2.9909	4.1851	0.7146
Bank 3	3.2648	4.1598	0.7848	3.6071	5.1417	0.7015
Bank 4	2.7052	2.8573	0.9468	1.7740	2.2455	0.7900
Bank 5	0.2670	0.3886	0.6870	0.3673	0.6868	0.5349
Bank 6	2.1257	3.0050	0.7074	2.9732	4.5735	0.6501
Bank 7	1.4309	1.9248	0.7434	1.4082	2.1262	0.6623
Bank 8	2.1251	2.6261	0.8092	2.1105	2.6112	0.8083
Bank 9	1.9529	2.6275	0.7433	2.0544	2.8547	0.7197
Bank 10	1.2857	1.7883	0.7190	1.6341	2.5488	0.6411
Bank 11	0.4305	0.5369	0.8017	0.2420	0.3235	0.7480
Bank 12	0.0402	0.0487	0.8259	0.0402	0.0487	0.8259
Bank 13	0.3531	0.4775	0.7395	0.3531	0.4775	0.7395
Bank 14	0.2303	0.3718	0.6195	0.3388	0.5625	0.6024
Bank 15	1.4896	2.2401	0.6650	1.8881	2.9103	0.6488
Bank 16	1.5687	2.3384	0.6709	2.3706	3.7069	0.6395
Bank 17	0.5762	0.7157	0.8050	0.5656	0.7047	0.8026
Bank 18	0.2632	0.3765	0.6990	0.2729	0.3934	0.6936
Bank 19	0.4211	0.6112	0.6890	0.4211	0.6112	0.6890
Bank 20	0.2365	0.3559	0.6645	0.3191	0.4930	0.6474

Note: This table gives the estimated deposit insurance premiums based on the parameter values displayed in Table 2. All the deposit insurance premiums have been multiplied by 1000. The second and third columns give the means of the estimated premiums using parameter values calculated from the FPT formulas. The symbols IPP and IPP^A are the per-monetary-unit deposit insurance premium considering no early bankruptcy and the early bankruptcy, respectively. The fourth column gives the means of the IPP / IPP^A ratios. The fifth to seventh columns give the corresponding estimates based on parameter values obtained from the RV formulas.

Table 3: Estimates of the deposit insurance premiums of per deposit amount (continuously)

	FPT formulas			RV formulas		
Symbol	IPP **($)**	IPP^A **($)**	Ratios (%)	IPP **($)**	IPP^A **($)**	Ratios (%)
Bank 21	0.2815	0.4222	0.6668	0.2815	0.4222	0.6668
Bank 22	0.1470	0.1977	0.7435	0.1470	0.1977	0.7435
Bank 23	0.7062	1.0025	0.7045	0.7059	1.0021	0.7045
Bank 24	17.7510	19.0310	0.9327	17.7510	19.0320	0.9327
Bank 25	0.7768	1.1438	0.6792	0.7879	1.1619	0.6781
Bank 26	0.4701	0.6248	0.7523	0.4701	0.6248	0.7523
Bank 24	0.4101	0.4920	0.8336	0.4301	0.5124	0.8393
Bank 28	2.3468	3.3577	0.6989	2.3469	3.3578	0.6989
Bank 29	0.5278	0.7857	0.6718	0.6917	1.0658	0.6490
Bank 30	2.1788	3.0827	0.7068	2.1788	3.0827	0.7068
Bank 31	1.9406	3.0260	0.6413	1.9793	3.1295	0.6325
Bank 32	6.2978	8.0104	0.7862	6.2978	8.0104	0.7862
mean	1.8932	2.3854	0.7404	1.9822	2.6209	0.7115

Note: This table gives the estimated deposit insurance premiums based on the parameter values displayed in Table 2. All the deposit insurance premiums have been multiplied by 1000. The second and third columns give the means of the estimated premiums using parameter values calculated from the FPT formulas. The symbols IPP and IPP^A are the per-monetary-unit deposit insurance premium considering no early bankruptcy and the early bankruptcy, respectively. The fourth column gives the means of the IPP / IPP^A ratios. The fifth to seventh columns give the corresponding estimates based on parameter values obtained from the RV formulas.

Table 4: Estimates involving risk-shifting behavior

	Debt/Asset value		*IPP*		IPP^A	
Symbol	b_1	p-value	b_2	p-value	b_2	p-value
Bank 1	-0.0875	0.5248	2.2334***	0.0000	2.7196***	0.0000
Bank 2	-1.4014	0.9200	1.6881***	0.0000	1.9855***	0.0000
Bank 3	0.0070	0.4971	1.7078***	0.0000	1.9982***	0.0000
Bank 4	-0.4512	0.9892	0.0338***	0.0003	0.0333***	0.0002
Bank 5	-4.6722	0.9625	0.1066 ***	0.0000	0.1139***	0.0000
Bank 6	-0.1294	0.5217	2.1321***	0.0000	2.7382***	0.0000
Bank 7	1.1635	0.2769	0.9053***	0.0000	1.0551***	0.0000
Bank 8	-1.3568	0.9662	0.0086	0.4189	0.0104	0.4213
Bank 9	-0.7942	1.0000	0.0047	0.2545	0.0049	0.3024
Bank 10	-10.8120	0.9949	0.3868**	0.0390	0.4579*	0.0651
Bank 11	-8.4711	0.9988	0.4058***	0.0000	0.4778***	0.0002
Bank 12	-6.7241	0.7777	0.0480**	0.0243	0.0572*	0.0258
Bank 13	5.4662	0.1875	0.8720***	0.0006	1.1485***	0.0006
Bank 14	-27.1490	0.9988	0.1753	0.1052	0.2462	0.1107
Bank 15	-1.2675	0.6768	1.4578***	0.0001	2.1416***	0.0001
Bank 16	3.6891	0.2561	3.0201***	0.0016	4.2831***	0.0014
Bank 17	-4.0592	0.9962	-0.0531	0.9291	-0.0671	0.9313
Bank 18	-14.3580	0.9985	0.2132**	0.0478	0.2886**	0.0483
Bank 19	-5.2989	0.7604	1.0720***	0.0069	1.5677***	0.0076
Bank 20	-7.6545	0.7318	0.7915***	0.0002	1.1956***	0.0002

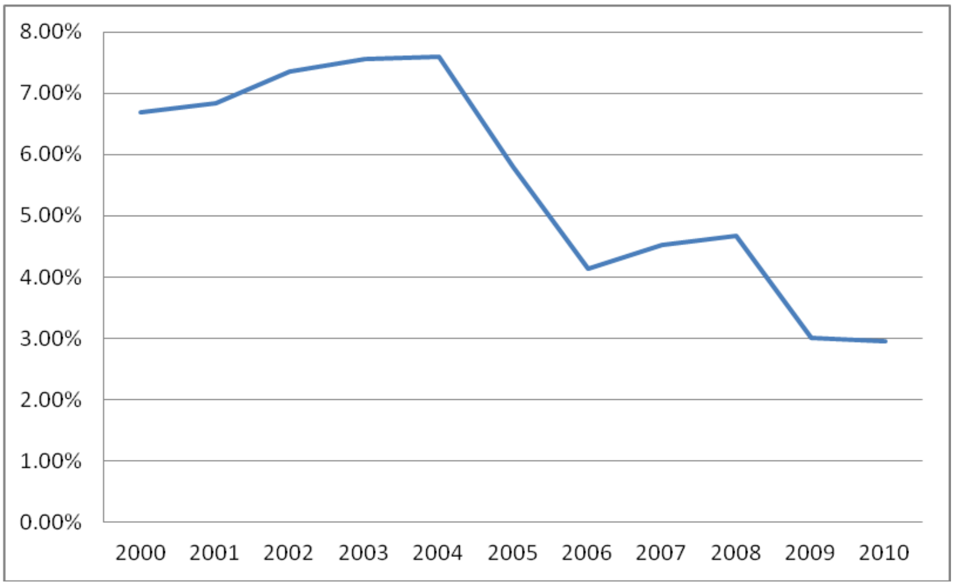
Note: This table gives estimates from Equations (21) and (22). The second and third columns give the estimates for parameter b_1 and their p-values from Equation (21). The fourth and fifth columns give the estimates for parameter b_2 and their p-values from Equation (22); these estimations are based on premiums calculated using Merton's model. The final two columns show the same b_2 estimates for p-values based on premiums calculated using our model. *** denotes significance at the 1% level, ** at the 5% level, and * at the 10% level.

Table 4: Estimates involving risk-shifting behavior (continuously)

	Debt/Asset value		*IPP*		IPP^{A}	
Symbol	b_1	p-value	b_2	p-value	b_2	p-value
Bank 21	-12.8600	0.9107	0.2971**	0.0236	0.3954**	0.0291
Bank 22	-29.3260	0.9966	0.0171	0.3656	0.0189	0.3881
Bank 23	-7.4620	0.8876	0.4122*	0.0553	0.5757*	0.0598
Bank 24	-0.6175	0.7183	0.7303**	0.0280	0.7824**	0.0280
Bank 25	-10.7550	0.8601	0.9813***	0.0072	1.4037***	0.0077
Bank 26	-11.0230	0.9961	0.0954	0.1962	0.1200	0.2012
Bank 24	-14.0950	0.8940	0.0550	0.3837	0.0722	0.3825
Bank 28	-7.6255	0.9774	0.5722*	0.0844	0.8147*	0.0845
Bank 29	-10.9360	0.9672	0.0741	0.3265	0.0962	0.3463
Bank 30	1.6636	0.4184	1.7545***	0.0088	2.4063**	0.0105
Bank 31	-0.2138	0.5308	2.5245***	0.0002	3.6635***	0.0002
Bank 32	-2.7621	0.9483	1.4166**	0.0210	1.5731**	0.0190
Significant Number	0		24		24	

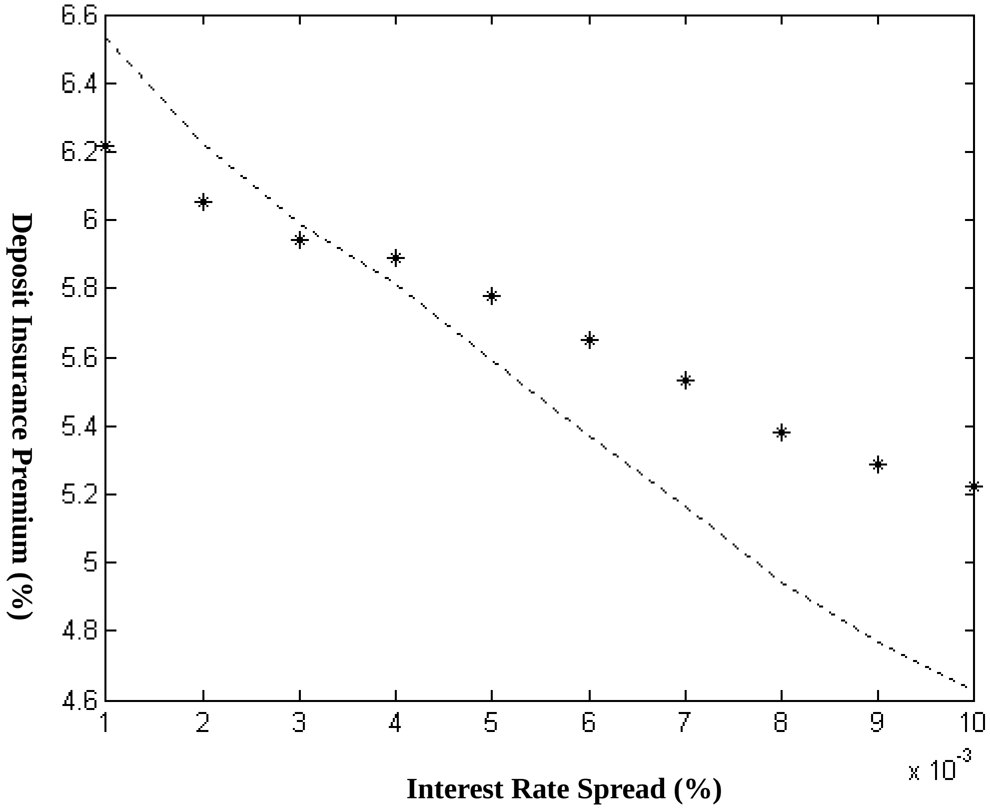
Note: This table gives estimates from Equations (21) and (22). The second and third columns give the estimates for parameter b_1 and their p-values from Equation (21). The fourth and fifth columns give the estimates for parameter b_2 and their p-values from Equation (22); these estimations are based on premiums calculated using Merton's model. The final two columns show the same b_2 estimates for p-values based on premiums calculated using our model. *** denotes significance at the 1% level, ** at the 5% level, and * at the 10% level.

Figure 1: The Trend of Interest Rate Spread in Taiwan Financial Markets



Note: This figure shows the trend of average interest rate spread for Taiwanese five top banks. The data is obtained from TEJ Databank.

Figure 2: Relationship between Deposit Insurance Premiums and Interest Rate Spreads



Note: The vertical axis denotes the deposit insurance premiums and the horizontal axis denotes the interest rate spreads, with Bank 1 as the example. The dividend yield for the bank is 0.16% and the interest rate spreads range from 0.1% to 1%. The broken line represents premiums calculated using the traditional model. The asterisk line represents premiums calculated using our model.